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Perturbative Dynamics of Fractional Strings on Multiply Wound D-strings

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Abstract

Fractional strings in the spectrum of states of open strings attached to a multiply wound D-brane is explained. We first describe the fractional string states in the low-energy effective theory where the topology of multiple winding is encoded in the gauge holonomy. The holonomy induces twisted boundary conditions responsible for the fractional moding of these states. We also describe fractional strings in world sheet formulation and compute simple scattering amplitudes for Hawking emission/absorption. Generalization to fractional DN-strings in a 1-brane 5-brane bound state is described. When a 1-brane and a 5-brane wraps Q_1 and Q_5 times respectively around a circle, the momentum of DN-strings is quantized in units of $2\pi/LQ_1Q_5$. These fractional states appear naturally in the perturbative spectrum of the theory.

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Dynamics of strings and D-branes [1, 2] have been a subject of interest recently. Perturbatively, such a system can be described using world sheets with Dirichlet boundary conditions [3, 4, 5, 6, 7, 8, 9]. Upon compactification, a new element enters the story: winding states of D-branes and strings. In [10, 11], a configuration where a D-brane wraps multiple times around a cycle was introduced. These configurations are motivated by S-duality [10] and are needed to account for the entropy of D-branes in the "fat black hole limit" [11]. Excitations of these branes have been referred to in the literature as "fractional strings." Their momentum is quantized in units of $2\pi/nL$ where n is the winding number of the D-brane and L is the period of the cycle. Quantization of momenta in fractional units can be motivated intuitively as fluctuations of a long string of length nL. They can also be understood as S-dual of excited states of ordinary strings [10], whose spectrum is given by

$$M = \sqrt{(nTL)^2 + 8\pi TN} = nTL + \frac{4\pi N}{nL} + \ldots \ .$$

Interesting interpretation for the spectrum of fractional strings in string field theory language was described in [12]. What is missing is a D-brane formulation of these fractional strings.

In this note, we explain how fractional strings arises naturally in the spectrum of states of open strings attached to a multiply wound D-brane. Taking doubly wound D-string as a concrete example, we first explain the fractional states in the spectrum of low energy effective theory. Then, we explain how the same physical spectrum follows from the spectrum of vertex operators in the world sheet sigma model. Using this language, we compute simple scattering amplitudes involving these fractional strings. Finally, we discuss generalizations to fractional states in D-branes with higher winding number and fractional DN-strings.

1 Multiply wound D-strings and Wilson-line Backgrounds

We begin by describing multiply wound D-branes. We will consider type IIB theory and compactify the X_1 direction into a circle of period L. To be concrete, let us focus on D-strings wound exactly twice. A closely related configuration carrying the same amount of Ramond-Ramond charge is a bound state of two D-strings, each winding once around the circle, and its spectrum includes a $U(2)$ Yang Mills gauge fields living on the D-string world volume [13]. As was explained in [14], these two configurations are distinguished by the presence of non-trivial gauge holonomy in the background. The bound state of two singly wound D-strings have the trivial holonomy

$$U = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Figure 1: Schematic illustration of bound state of two singly wound D-strings and a long D-string wound twice.

whereas a single long string wound twice have the non-trivial holonomy

$$U = \left(\begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array} \right).$$

This makes perfect sense, as it means charges on one branch of the doubly wound string returns to the other branch as it parallel transports around the cycle once, and returns to the original branch after parallel transporting around the cycle once more.

Gauge holonomies are due to Wilson lines and are related by

$$U = P[e^{i \oint A}].$$

The main point then is that multiply wound D-string can be studied perturbatively by expanding around a vacuum with a Wilson line background.

2 Fractional strings in low-energy effective action

In this section, we describe how the spectrum of fractional string states arises in the low energy effective theory. The low energy effective theory on the world volume of D-string with $Q_1 = 2$ is the $U(2)$ supersymmetric Yang Mills theory on $d = 2$ with adjoint matter hypermultiplets.

$$S = \mathrm{Tr}\left((\partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu])^2 + (\partial_\mu \Phi_M + [A_\mu, \Phi_M])^2 + [\Phi_M, \Phi_N]^2 \right) \tag{1}$$

Indecies μ and ν run over world volume dimensions $\{01\}$ and M and N run over transverse dimensions $\{23456789\}$. In order to specify the configuration where one long string is wound twice, we set the Wilson line background to

$$U = P[e^{i \oint A}] = \left(\begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array} \right).$$

It will be convenient to work in Coulomb gauge. In this gauge, A_1 will be constant along X_1. Turning on a Wilson line background is seen to be equivalent to setting the vacuum expectation value of A_1 to

$$\langle A_1 \rangle = \Lambda = \frac{\pi}{2L} \left(\begin{array}{cc} -1 & 1 \\ 1 & -1 \end{array} \right)$$

Due to the presence of commutator term in the action (1), vacuum expectation value of A_1 will give rise to a non-standard kinetic term. This can fixed by invoking a well known relation which exchanges Wilson line backgrounds with non-trivial boundary conditions. Consider a gauge function

$$g(X_1) = e^{i\Lambda X_1}$$

and define a_μ and ϕ_M by relation

$$\begin{aligned} A_\mu &= (\partial_\mu g) g^{-1} + g\, a_\mu\, g^{-1} \\ \Phi_M &= g\, \phi_M\, g^{-1} \end{aligned} \tag{2}$$

Since $g(X)$ is not globally defined (it is not single valued under $X \to X + L$), this is not a gauge transformation. It should instead be thought of as a field redefinition.

The form of the action is unchanged by the field redefinition

$$S = \mathrm{Tr}\left((\partial_\mu a_\nu - \partial_\nu a_\mu + [a_\mu, a_\nu])^2 + (\partial_\mu \phi_M + [a_\mu, \phi_M])^2 + [\phi_M, \phi_N]^2 \right).$$

Vacuum expectation value of a_μ vanish, so the kinetic term of this action is standard, and the Wilson line is trivial. In exchange, the field ϕ_M acquires a non-trivial boundary condition as it goes around the circle.

$$\phi_M(X_1 + L) = U^{-1}\, \phi_M(X_1)\, U.$$

Now we can understand the spectrum and the vacuum structure in the matter sector. Let us expand the matter hypermultiplet into components of $U(2)$ adjoint representations

$$\phi_M = \phi_M^0 \sigma^0 + \phi_M^1 \sigma^1 + \phi_M^2 \sigma^2 + \phi_M^3 \sigma^3.$$

In terms of these components, the boundary condition reads

$$\phi^0(x + L) = \phi^0(x), \qquad \phi^1(x + L) = \phi^1(x)$$

$$\phi^2(x + L) = -\phi^2(x), \qquad \phi^3(x + L) = -\phi^3(x),$$

that is, ϕ^0 and ϕ^1 are periodic and ϕ^2 and ϕ^3 are antiperiodic. The periodic fields are integrally moded while anti-periodic fields are half-integrally moded. The combined spectrum is quantized in units of $2\pi/nL$ which is the spectrum of fractional strings we set out to find.

Vacuum structure in the matter sector is determined by the expectation value of the zero modes. Since ϕ^2 and ϕ^3 are antiperiodically moded, these fields do not have zero modes. The ϕ^0 simply corresponds the center of mass. The vacuum is parameterized by the zero modes of ϕ^1 which takes value on R/Z_2.

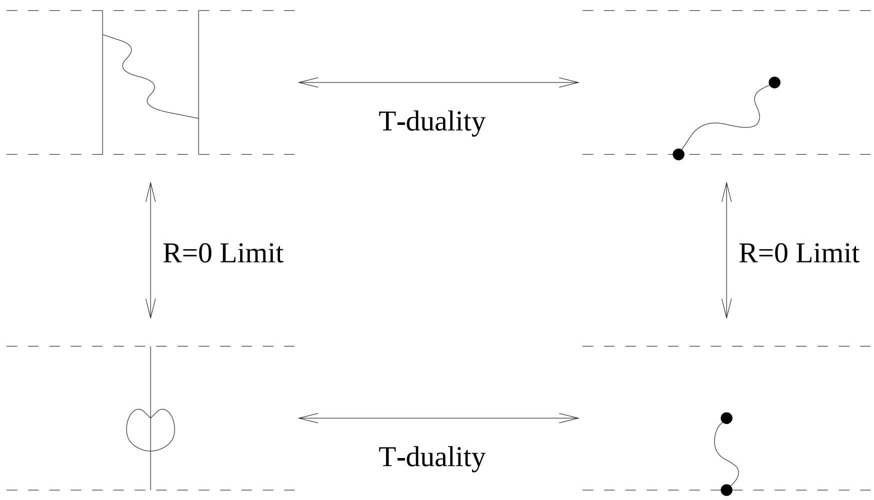

Figure 2: Stringy illustration of fractional strings and its T-dual

When ϕ^1 has a non-vanishing expectation value, ϕ^2 and ϕ^3 acquire a mass and the fractional strings disappear from the low-energy spectrum. Only ϕ^0 and ϕ^1 survive in the low-energy spectrum. Physical interpretation of this configuration becomes more clear if we diagonalize ϕ^1 by performing a global $U(2)$ transformation. In this basis, the vacuum expectation value of A_1 becomes

$$A_1 = \frac{1}{L}\begin{pmatrix} 0 & 0 \\ 0 & \pi \end{pmatrix} \tag{3}$$

Turning on the vacuum expectation value of ϕ^1 then corresponds to separating a pair of singly wound D-string. The gauge group $U(2)$ is broken to $U(1) \times U(1)$ subgroup which corresponds to gauge fields living on each of the branes. The vacuum expectation value of these $U(1)$ gauge field can be read off from the diagonal of (3). These vacuum expectation values specifies a unique superposition of fundamental winding states bound to the D-branes as was described in [13]. This also provides a concrete physical picture for the higgsing of the fractional states. The fractional states corresponds to strings stretched between the two wound D-strings with the above $U(1)$ expectation values. They become light when the two D-strings coincide. The fact that these states are fractional can be most easily seen by T-dualizing along the X_1 direction. Under T-duality, the period of the cycle is inverted and a D-string becomes a zero brane. Since

$$4\pi A_1 = \frac{4\pi}{L}\begin{pmatrix} 0 & 0 \\ 0 & \pi \end{pmatrix} = \frac{L'}{2}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

where L' is the dual period, this configuration corresponds to that of a two zero branes separated by half a period [14] (see figure 2). This was anticipated in [15] but here we

derived it directly using the low energy effective description of the D-brane. The open string stretching from one zero brane to the other must then wrap around the cycle by half integral multiple of the period. T-dualizing back to a D-string picture, we find that the momentum of these states are half integrally moded, which is precisely what we expect for the fractional strings.

3 World sheet description of multiply wound D-strings

In this section, we extend the result of the previous section to full string theory. To this end, it suffices to consider an open string non-linear sigma model in a Wilson line background. The spectrum of fractional strings can be read off from the spectrum of marginal operators of this sigma model. Once the vertex operators are constructed, correlation functions can be computed in a straightforward manner.

The basic structure of non-linear sigma models in a Wilson line background is quite simple. One simply adds a coupling to background gauge field in the Polyakov path integral.

$$\langle \ldots \rangle = \mathrm{Tr}[P \int DX(\ldots)\, e^{-S+i\oint A_1 \partial_t X_1}] \tag{4}$$

Processes involving fractional strings are sometimes easier to visualize in the T-dual description where the string winds by a fractional period. The T-dual of the above path-integral expression is

$$\langle \ldots \rangle = \mathrm{Tr}[P \int DX(\ldots)\, e^{-S+i\oint A_1 \partial_n X_1}] \tag{5}$$

Care is needed to path-order the exponential as the gauge field A is non-abelian. This only affects the vertex operators sitting at the boundary of the world sheet. Of course, fractional strings are such operators. To be concrete, let us consider computing an amplitude involving two boundary vertex operators and arbitrary number of bulk vertex operators. Due to path ordering, the correlation function takes the form

$$\langle V_1(\sigma_1)V_2(\sigma_2)(\ldots)\rangle = \int DX\,(\ldots)\,\mathrm{Tr}[V_1(\sigma_1)U(\sigma_1,\sigma_2)V_2(\sigma_2)U(\sigma_2,\sigma_1)\,]\,e^{-S}$$

where

$$U(\sigma_1,\sigma_2) = P[e^{i\int_{\sigma_2}^{\sigma_1} A_1\partial_n X_1}]$$

The only non-trivial effect of $U(\sigma_1,\sigma_2)$ is its action on the Chan-Paton factor of $V(\sigma)$. Since we are interested in the physics of fractional strings, let us set the Chan-Patton factors $T_1 = T_2 = \sigma^3$ and examine the expression

$$\mathrm{Tr}[T_1 U(\sigma_1,\sigma_2) T_2 U(\sigma_2,\sigma_1)]$$

more closely. Just as in the previous section, it is convenient to work in Coulomb gauge where $A_1 = \Lambda =$ constant. Using the fact that

$$\sigma^3 \Lambda \sigma^3 = \frac{\pi}{2L} \begin{pmatrix} -1 & -1 \\ -1 & -1 \end{pmatrix}$$

one can easily show that

$$\mathrm{Tr}[T_1 U(\sigma_1, \sigma_2) T_2 U(\sigma_2, \sigma_1)] = e^{-i\int_{\sigma_2}^{\sigma_1} \frac{\pi}{L}\partial_n X} + e^{-i\int_{\sigma_1}^{\sigma_2} \frac{\pi}{L}\partial_n X}.$$

In the electrostatic language, this corresponds to placing an electric dipole with its moment normal to the boundary of the world sheet, which has precisely the effect of turning on an electrostatic potential difference between the two components of the world sheet boundary marked by σ_1 and σ_2. This is precisely the expected world sheet description of string states which stretches between two 0-branes separated by distance $L'/2 = (2\pi)^2/L$.

Now let us examine the stringy dynamics of these fractional winding states. To this end we must construct vertex operators for these states and compute their correlation function. An essential ingredient in such a vertex operator is the piece which encodes the fact that the string is stretched between separated D-branes. This can be determined from the singularity structure in the electric field at the boundary changing point as follows. First consider a strip $0 \le \mathrm{Im}[\sigma] \le \pi$ with boundary condition $X(\sigma) = 0$ at $\mathrm{Im}[\sigma] = 0$ and $X(\sigma) = V$ at $\mathrm{Im}[\sigma] = \pi$. The background $X(\sigma)$ which satisfies this boundary condition is

$$X(\sigma) = \frac{V}{\pi}\mathrm{Im}[\sigma].$$

Now map this strip to a half-plane using the map $z = e^{\sigma}$. This maps the boundary changing points to $\sigma = 0$ and $\sigma = \infty$. In the z-plane,

$$X(z, \bar{z}) = \frac{V}{2\pi i}(\ln(z) - \ln(\bar{z}))$$

so near $z = 0$,

$$\partial X(z) = \frac{V}{2\pi i}\frac{1}{z}$$

and

$$\bar{\partial}\bar{X}(\bar{z}) = -\frac{V}{2\pi i}\frac{1}{z}$$

This is precisely the structure of operator product expansion

$$\partial X(z) e^{iqX} e^{-iq\bar{X}}(0) = \frac{2iq}{z} e^{iqX} e^{-iq\bar{X}}(0)$$

$$\bar{\partial}\bar{X}(\bar{z}) e^{iqX} e^{-iq\bar{X}}(0) = -\frac{2iq}{\bar{z}} e^{iqX} e^{-iq\bar{X}}(0)$$

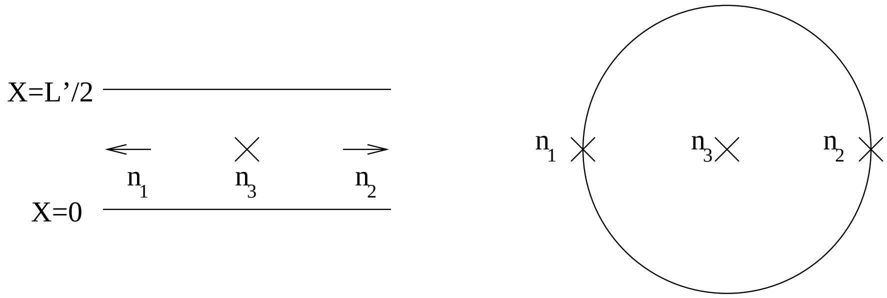

Figure 3: String world sheet amplitude for a process where a wound closed string with momentum q and polarization ε_{MN} is absorbed by one of the zero branes, creating a pair of fractionally wound open string states with winding numbers p_1 and p_2.

for

$$q = -\frac{V}{4\pi} = -\frac{L'}{8\pi} = \frac{\pi}{L}.$$

On the boundary, $X = -\bar{X}$, so

$$e^{iqX}e^{-iq\bar{X}} = e^{i\,2qX}.$$

which effectively shifts the winding number by q. By now the basic construction of the vertex operator for the fractional string state is clear. World sheet supersymmetry constrains the form of the vertex operator to take the conventional form

$$V(z) = (\partial X^\mu + i(2k\cdot\psi)\psi^\mu)e^{i\,2kX}(z)$$

but the boundary condition shifts the winding number

$$n = \frac{4\pi}{L'}k_1$$

to take on half-integral values. So we see how fractional string spectrum arises from the world sheet considerations. These vertex operators can be used to compute amplitudes involving fractional strings. Perhaps the simplest such amplitude is a process where a macroscopic string [16] in a ground state of winding number n_3 with momentum $\vec{q}$ is absorbed by one of the zero branes, creating a pair of fractionally wound open string states with winding numbers n_1 and n_2. (see figure 3.) The amplitude takes the general form

$$A = \int \frac{dz_1\, dz_2\, d^2z_3}{V_{CKG}} \langle V_1(z_1)V_(z_2)V_3(z_3,\bar{z}_3)\rangle$$

where

$$\begin{aligned} V_1(z_1) &= \xi^1_M(\partial X^M + i(2p_1\cdot\psi)\psi^M)e^{i\,2p_1X}(z_1) \\ V_2(z_2) &= \xi^2_M(\partial X^M + i(2p_2\cdot\psi)\psi^M)e^{i\,2p_2X}(z_2) \end{aligned}$$

for fractional strings and

$$V_3(z_3,\bar{z}_3) = \varepsilon_{MN}\,(e^{-\phi}\psi^M e^{iq_L X}(z_3))\,(e^{-\bar{\phi}}\bar{\psi}^N e^{iq_R\bar{X}}(\bar{z}_3)).$$

for the macroscopic string with

$$\begin{aligned} p_1 &= (p_1^0, \frac{L'}{4\pi}n_1) \\ p_2 &= (p_2^0, \frac{L'}{4\pi}n_2) \\ q_L &= (q^0, \frac{2\pi}{L'}m_3 + \frac{L'}{4\pi}n_3, \vec{q}) \\ q_R &= (q^0, \frac{2\pi}{L'}m_3 - \frac{L'}{4\pi}n_3, \vec{q}) \end{aligned}$$

and $m_3 = 0$. This is precisely of the form of the amplitude computed in [6] so we simply quote the result

$$A = \frac{\Gamma(-2t)}{\Gamma(1-t)^2}t^2\left(\xi^1\!\cdot\!\varepsilon\!\cdot\!\xi^2 + \xi^2\!\cdot\!\varepsilon\!\cdot\!\xi^1\right)$$

where

$$t = -2p_1\!\cdot\!p_2$$

with the requirement that n_1 and n_2 take on half-integer values. Of course, by T-duality, this amplitude is exactly equivalent to the amplitude for a process where a graviton of momentum q is absorbed by a doubly wound D-string creating a pair of fractional momentum open strings with momentum p_1 and p_2.

4 Generalizations to multiply wound DD and DN strings

Until this point we have focused on fractional excitations of D-string wound exactly twice. The basic structure underlying the fractional states are the $U(2)$ holonomy matrix

$$U = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

and its action on the $U(2)$ generators

$$\mathrm{Tr}[TU^{-1}TU] = \pm 1.$$

The sign of $\mathrm{Tr}[TU^{-1}TU]$ induced the twisting of the boundary condition which gives rise to fractionally moded states in the low-energy dynamics, and was ultimately responsible for the placement of 0-branes separated by half a period in the T-dual description.

This structure has a natural generalization to configurations with higher winding numbers. The low-energy effective theory for the n-tuply wound D-string configuration is a $U(n)$ gauge theory with the holonomy

$$U = \begin{pmatrix} 0 & 1 & & \\ & 0 & \ddots & \\ & & 0 & 1 \\ 1 & & & 0 \end{pmatrix}. \tag{6}$$

Twisting of the boundary is induced by the phase factor given by the eigenvalues of the matrix

$$M_{ab} = \mathrm{Tr}[T_a U^{-1} T_b U].$$

It turns out that for all n, the spectrum of eigenvalues is precisely the n-fold degenerate spectrum of

$$\left\{1,\ e^{2\pi i/n},\ e^{4\pi i/n},\ \ldots,\ e^{2(n-1)\pi i/n}\right\} \tag{7}$$

which implies a combined spectrum quantized in units of $2\pi/nL$.

To see that the spectrum of M_{ab} is indeed of the form (7), simply note that adjoint representation of $U(n)$ is a tensor product $n \otimes \bar{n}$. Natural basis for the fundamental representation of $U(n)$ in this context the eigenvectors v_a of the holonomy matrix U with eigenvalues $\lambda_a = \exp(2\pi i a/n)$. An adjoint element $v_a \otimes \bar{v}_b$ transforms under U like

$$U^{-1}(v_a \otimes \bar{v}_b)U = e^{2\pi i(a-b)/n}(v_a \otimes \bar{v}_b)$$

where a and b takes integer values between 0 and $n-1$. Now, $(a-b)$ mod n is an n-fold degenerate set of integers ranging from 0 to $n-1$. Perhaps an example would suffice to make this point. For $n=3$, $((a-b)$ mod $n)$ takes on values

	b=0	b=1	b=2
a=0	0	2	1
a=1	1	0	2
a=2	2	1	0

as is expected.

We can generalize these ideas further to fractional states in the Dirichlet-Neumann (DN) sector. DN strings appear when branes of different dimensionality, say a 1-brane and a 5-brane, are simultaneously present. The 1-brane 5-brane system compactified on T^5 is by now familiar as a model of extremal black hole with a regular horizon [17, 18]. When the 5-brane wraps around T^5, it is possible in principle to turn on non-trivial holonomies for all cycles.

Here, we will restrict our attention to the simple case where the only non-trivial holonomy is along the direction parallel to the 1-brane. This is essentially the case considered in [11].

Now consider a 1-brane wrapped Q_1 times and a 5-brane wrapped Q_5 times around the cycle in X_1 direction. This corresponds to turning on a holonomy matrix $U_{(1)}$ of $U(Q_1)$ and $U_{(5)}$ of $U(Q_5)$ of the type (6).

The DN-strings carries a Chan-Patton factor $T_{(1,5)}$ which transforms as $Q_1 \otimes \bar{Q}_5$ and computing the extent of twist in the boundary condition reduces to computing the eigenvalue of

$$M_{ab} = \mathrm{Tr}[(T_{(1,5)})^\dagger U_{(1)} T_{(1,5)} U_{(5)}^{-1}].$$

Just as in the previous case, it is convenient to construct the basis $T_{(1,5)}$ using the tensor product of v_a and $\bar{w}_b$ where v_a is an eigenvector of $U_{(1)}$ with eigenvalues $\exp(2\pi i a/Q_1)$ and w_b is an eigenvector of $U_{(5)}$ with eigenvalues $\exp(2\pi i b/Q_5)$. Then

$$U_{(1)}(v_a \otimes w_b)U_{(5)}^{-1} = e^{2\pi i(aQ_5 - bQ_1)/Q_1Q_5}(v_a \otimes w_b)$$

Just as in the previous case, let us make a table of $((aQ_5 - bQ_1) \bmod Q_1Q_5)$ for $Q_1 = 2$ and $Q_5 = 3$.

	b=0	b=1	b=2
a=0	0	4	2
a=1	3	1	5

We see that $((aQ_5 - bQ_1) \bmod Q_1Q_5)$ takes on integer values between 0 and $Q_1Q_5 - 1$ with no degeneracy. This will be true in general for Q_1 and Q_5 relatively prime, implying that the spectrum of these states will be quantized in units of $1/Q_1Q_5$ which is precisely what was anticipated in [11]. (In general, the spectrum will be quantized in units of N/Q_1Q_5 with degeneracy N where N is the greatest common divisor of Q_1 and Q_5.) Perturbative dynamics of fractional DN-strings can then be computed along the lines of [7].

5 Conclusions

In this article we considered the excitations on D-branes wrapped multiple times by turning on a background with a non-trivial gauge holonomy. We find a spectrum of states quantized in fractional units of momentum along the compactified direction. These are the so-called "fractional strings" whose existence have been postulated previously on the grounds of entropy counting and S-duality. We have provided an explicit check on this postulate by constructing these states in the D-brane formulation.

In the low-energy effective theory, gauge holonomies have the effect of twisting the boundary condition. Fractional states arises naturally as a result of twisted boundary conditions.

Same basic conclusion can be drawn by considering the full open-string theory in the world sheet formulation. Here, it was convenient to consider the T-dual of multiply wound D-string which corresponded to n D0-branes equally spaced along the circle of compactification. The fractional momentum states have a natural interpretation in the T-dual picture as strings stretching between these D0-branes which winds around the period of compactification in fractional units. Vertex operators for these twisted strings were constructed with which we computed the Hawking emission/absorption amplitude.

Finally, we described how the same idea can be used to explain the fractional spectrum of DN-strings. For example, it is possible to show that the momentum of DN-strings in a 1-brane 5-brane system with winding numbers Q_1 and Q_5 are quantized in units of $1/Q_1Q_5$. This has been anticipated in [11] but it is pleasant to find an explicit D-brane construction.

Recent exciting work on string dualities has predicted the existence of exactly one threshold bound state [19] in the quantum mechanics of these D-branes [20, 21, 22]. By construction, our discussion of fractional strings is perturbative, and at leading order captures only the semi-classical aspects of D-brane dynamics. Presumably, the full quantum dynamics of D-branes such as the existence of these bound states is encoded in the full re-summation of scattering amplitudes, but it is not clear how this is done in detail. Perhaps something can be learned by examining the large order growth [23] of the perturbative expansion. It would be very interesting to understand how perturbation theory and D-brane quantum mechanics fit together in the full treatment of non-perturbative string dynamics. In particular, we have shown in this article that the semi-classical configuration of a D-string wound n times on period L is T-dual to D0-branes equally spaced along the dual period L'. Recently, it was argued on the grounds of S-duality that the threshold bound state approximates this semi-classical configuration in the limit where period L' is small [15]. Something interesting appears to be happening when L' is taken small enough to squeeze the 0-brane bound state wave function to a size smaller than its natural scale. It would be very interesting to understand this phenomenon in detail by studying the dynamics of D0-branes along the lines of [20, 21, 22] but on a compactified space. We leave these fascinating questions for future investigations.

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